Kondo physics with single electron transistors

M.A. Kastner⁠,*, D. Goldhaber-Gordon⁠†

⁠*Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
⁠†Harvard University, Department of Physics and Society of Fellows, 17 Oxford Street, Cambridge, MA 02138, USA

Abstract

The coupling between the confined droplet of electrons and the leads in a single electron transistor (SET) has made it possible to explore Kondo physics in ways that were never before possible. In particular, whereas the energy of the localized spin is fixed for the traditional Kondo system, a magnetic impurity in a metal, it can be varied in an SET, allowing detailed comparisons with the theory for the equilibrium Kondo problem. Furthermore, one can apply a finite bias voltage between the leads revealing non-equilibrium Kondo effects as well. © 2001 Elsevier Science Ltd. All rights reserved.

PACS: 75.20.Hr; 73.23.Hk; 72.15.Qm; 73.23.—b

Keywords: A. Nanostructures; D. Electron transport; D. Kondo effects

1. Introduction

When a droplet of electrons is confined to a small region of space and coupled only weakly to its environment, the number of electrons and their energy become quantized. In this way, the droplet behaves like an artificial atom. Specifically, there is an energy cost for adding an extra electron or for promoting confined electrons to higher-lying energy levels [1,2]. In fact, in a single electron transistor (SET), in which an artificial atom is coupled to conducting leads, the Anderson model, designed to explain the coupling of natural atoms to metals, provides a quantitative description of the coupled electronic system [3–5].

The Anderson model was developed to address one of the most challenging problems of 20th century physics: the behavior of a metal containing a magnetic impurity. At high temperatures the spin of the impurity is independent of the spins of the electrons in the metal, but at low temperatures many-body correlations lead to a singlet state, in which the spin of the impurity is screened by those of the conduction electrons. The incipient formation of this singlet as the temperature is lowered results in strong scattering of the conduction electrons near the Fermi energy and a consequent increase in the resistance. How the singlet state evolves with temperature and the consequences of this evolution for magnetization and conductivity is called the Kondo problem.

Because the coupling between the localized electron and those in the metal is strong for electronic states with a range of energies throughout the conduction band, the Kondo problem could not be solved with perturbation theory but required the invention of renormalization group theory. Indeed, Wilson points out the importance of the Kondo problem in his Nobel address [6,7]. However, when theory became adequate to account for the experiments on magnetic impurities in metals, it also made predictions that could not be tested with those systems. In particular, there were several parameters in the Anderson Hamiltonian to which the theory was very sensitive. For magnetic impurities these were determined by the properties of the elements involved and were, therefore, fixed for a particular chemical composition. It appeared, therefore, that some aspects of the theory could never be verified.

Beginning in the late 1980s, theorists proposed that the Kondo effect should also arise in nanometer-sized structures that allow tunneling between localized states and metal leads [3,4]. They argued that when a localized state contain an unpaired electron, it would be analogous to a magnetic impurity, and the electrodes would be analogous to the surrounding metal. It was predicted that because scattering would increase rather than reduce transport for tunneling, the Kondo effect would increase the conductance instead of the resistance at low temperature. With this modification, the powerful theoretical machinery that was
invented to explain magnetic impurities in metals could be used to predict the conductance in resonant tunneling through localized states.

Following the invention of SETs, detailed predictions were made about the modification of their characteristics by the Kondo effect [5,8]. However, many years passed without experimental realization of the phenomenon. The first clear demonstration was the work of Goldhaber-Gordon et al. [9], who made the effect dramatic by making smaller SETs than ever before. Soon thereafter several other groups observed Kondo phenomena in larger SETs [10–13]. It now appears that Kondo physics occurs very often in nanostructures. It has been seen in SETs that are relatively large as well as small; it has been seen for the standard case, in which the degeneracy causing the effect results from spin 1/2, as well as for cases in which the degeneracy is threefold [14]; and it has been seen in carbon nanotubes [15] as well as semiconductor nanostructures.

In this review we summarize our group’s work on the Kondo effect [9,16] and point out where further research is likely to be valuable. In Section 2, we summarize how we make our SETs and how they operate. In Section 3, we present the salient results of Kondo experiments. Finally, in Section 4, we briefly discuss some outstanding problems.

2. SET fabrication and function

We create the SET in a two-dimensional electron gas (2DEG) just below the surface of a GaAs/AlGaAs heterostructure. Electrodes on the surface deplete the 2DEG and narrow constrictions between electrodes form tunnel barriers. The device, for which the electrode structure is shown in Fig. 1(a), is made with a shallow (~20 nm deep) 2DEG and narrow (~20 nm) electrode features, created by electron beam lithography. The electrodes confine a droplet of electrons to a small region of space, which we estimate to

![Fig. 1.](image1)

![Fig. 2.](image2)
be about 100 m in diameter. The 2DEG regions outside the electrodes serve as source and drain electrodes and the middle (plunger) electrode serves as a gate.

The conductance is measured by applying a very small voltage \( V_{\text{dr}} \) between drain and source, small enough that the current is proportional to \( V_{\text{dr}} \). For current to flow the number of electrons on the droplet must fluctuate, say between \( N \) and \( N + 1 \). Thus, the \( N \)-th peak in the conductance occurs when the state of the droplet containing \( N \) electrons is in equilibrium with the state containing \( N + 1 \) electrons. Were the gate the only electrode contributing to the electrostatic energy of the droplet, the gate voltage at which the \( N \)-th peak occurred multiplied by the charge of the electron \( e \) would be the energy difference between the two states. However, since there are several electrodes near the droplet, the energy change caused by \( V_g \) is, instead, \( \alpha e V_g \), where \( \alpha = C_g/C \) is the ratio of the gate capacitance to the total capacitance. Therefore, a conductance peak occurs when

\[
\alpha e V_g(N) = E(N + 1) - E(N),
\]

apart from a constant, where \( E(N) \) is the total ground state energy of the droplet with \( N \) electrons.

In the classical Coulomb blockade model of the SET, the energy of the droplet containing \( N \) electrons is simply \((N e)^2/2C\), so that Eq. (1) predicts conductance peaks spaced equally in \( V_g \). Approximately periodic peaks are, indeed, observed when the coupling between the droplet of electrons and the leads is weak, and for this case the conductance increases and decreases by several orders of magnitude almost periodically as \( V_g \) is varied (Fig. 2). A calculation of the capacitance between the gate electrode and the droplet of confined electrons shows that the voltage between two peaks or two valleys is just that necessary to add one electron to the droplet. The name ‘single electron transistor’ comes from the observation that the transistor turns on and off again every time a single electron is added to it. This behavior is a direct consequence of charge quantization that results from localization. In addition to charge quantization, energy quantization is important when electrons are confined to small volumes. In addition to the energy \( U \) to add an extra electron to the artificial atom, there is a typical level spacing \( \Delta \varepsilon \) necessary to excite the artificial atom while keeping the number of electrons fixed. Furthermore, the coupling of the artificial atom to the leads gives rise to a typical width \( \Gamma \) of the conductance peaks at temperatures for which \( k_BT \ll \Gamma \). The level width is caused by lifetime broadening, because an electron in a level on the artificial atom can tunnel into the leads. Alternatively, one can say that the eigenstates of the system are mixtures of localized states on the artificial atom and extended states in the leads. We vary \( \Gamma \) by adjusting the voltages on the gates that form the constrictions.

When \( \Gamma \) increases, \( U \) decreases \([17]\), and we find that the conductance peaks now occur in pairs as shown in Fig. 2. More of these paired peaks are shown in Fig. 3. For an SET in which the artificial atom is sufficiently small and has low symmetry, all spatial degeneracies are lifted. For this case one would expect that increasing \( V_g \) would cause electrons to enter successively higher-lying energy levels, two electrons (of opposite spin) for each level. Although each level can accommodate two electrons, the Coulomb interaction results in an energy cost \( U \) to add a second electron when a level is singly occupied. Thus, one expects the typical energy to add an electron to alternate between \( U \) and \( U + \Delta \varepsilon \). As we show below, the pairing of peaks in Figs. 2 and 3 is also strongly affected by the Kondo effect.

![Fig. 3. Conductance vs plunger gate voltage \( V_g \) showing three sets of paired peaks.](image-url)
Fig. 4. Conductance vs plunger gate voltage $V_g$ (or $\epsilon_0/\Gamma$) at various temperatures. The vertical dashed lines mark gate voltages at which two charge states are degenerate. Between the dashed lines the number of electrons confined in the SET is odd, and the Kondo effect enhances the conductance. The inset shows the width vs temperature for one of the peaks at high temperatures; the extrapolation to $T = 0$ gives $\Gamma$.

Nonetheless, the two closely spaced peaks in a pair result from the same localized spatial state, and we next focus on one such pair of peaks.

3. Results of Kondo experiments

The theory of the SET, including the Kondo effect, is based on the Anderson model, in which the SET is approximated as a single localized state coupled by tunneling to two electron reservoirs [5]. The state can contain zero, one, or two electrons. Adding the first electron takes an energy $\epsilon_0$ referenced to the Fermi energy in the leads, but the second electron requires $\epsilon_0 + U$ [Fig. 1(b)]. Tunneling of electrons from the localized state into the leads gives rise to broadening of the localized-state energies, with full width $\Gamma$. Making the voltage $V_g$ on a nearby electrode [the plunger gate in Fig. 1(a)] more negative increases $\epsilon_0$. Thus, with a

Fig. 5. Conductance versus temperature for various values of $\epsilon_0$ on the right side of the left-hand peak in Fig. 4. The solid curve represents the results of numerical renormalization-group calculations. By fitting the latter we extract $T_K$ and $G_0$ for each set of data.
change in \( V_g \) one can change from the empty orbital regime, \( \epsilon_0 > 0 \), to the mixed valence regime, \( \epsilon_0 \sim 0 \), to the Kondo regime, \( \epsilon_0 < 0 \).

As mentioned above, current flow is possible only when two charge states of the droplet are degenerate, i.e. \( \epsilon_0 = 0 \) or \( \epsilon_0 + U = 0 \). Two of these charge degeneracy points occur at the values of \( V_g \) marked by vertical dashed lines in Fig. 4; the left degeneracy point corresponds to a change from even to odd occupancy of the level, and the right one to a change from odd to even. At high temperatures all conductance peaks broaden with increasing \( T \), as seen in Figs. 2 and 4. This provides a conversion between gate voltage and energy, allowing us to determine \( \epsilon_0 \) and \( U \) from the peak positions. Furthermore, a plot of width vs \( T \) gives a straight line that can be extrapolated to \( T = 0 \) to give \( \Gamma \) (see the inset of Fig. 4).

Thus, as \( T \) decreases, the reduction in thermal broadening results in a decrease in conductance between the peaks. However, theory predicts that at sufficiently low \( T \) the conductance increases again because of the Kondo effect. The coupling between the localized spin and the spins of the electrons in the leads gives rise, at \( T = 0 \), to a sharp peak in the density of states at the Fermi energy. This peak is illustrated in Fig. 1(b). There is one such peak for each lead, resulting in a set of states at the Fermi level strongly coupled from one lead, through the localized state, to the other lead. At \( T = 0 \), this coupling is predicted to give rise to perfect transmission (for symmetric coupling to the two leads) and,
consequently, a conductance equal to $2e^2/h$ [4]. As $T$ is increased, the conductance is predicted to decrease, approximately logarithmically, when $T \sim T_K$, where the Kondo temperature $T_K$ is a measure of the binding energy of the singlet. This qualitative behavior can be seen in Figs. 2 and 4.

In fact, the theory predicts that the temperature dependence of conductance for any $V_e$ between, but not too close to, the charge degeneracy points is universal at low $T$ [18]. Indeed, we find that all data in this region collapse onto the curve predicted by numerical renormalization group theory, as shown in Fig. 5. The data have been scaled by the Kondo temperature $T_K$ and the zero-temperature extrapolation of the conductance $G_0$. To carry out the fits we have constructed an empirical form [16] that provides a good fit to the numerical renormalization group calculations of Costi and Hewson [18].

As shown in Fig. 6, $T_K$ and $G_0$ determined in this way agree strikingly well with the predictions of theory. For $\varepsilon_0 \ll \Gamma$ scaling theory [19] predicts

$$T_K = \frac{\sqrt{U \Gamma}}{2} e^{\pi \varepsilon_0 (U + U)/U}$$  \hspace{1cm} (2)

Since $\varepsilon_0$, $U$, and $\Gamma$ are determined independently, $T_K$ is, in principle, predicted with no adjustable parameters. In fact, the exponent in Eq. (2) is in excellent agreement with the prediction, but the prefactor differs by a factor $\sim 3$, which we consider good agreement, given the sensitivity to the exponent. We have adjusted the prefactor for Fig. 6.

The zero temperature extrapolation of the conductance $G_0$ agrees with the form predicted from the Friedel sum rule [8]. Theory predicts that $G_0$ approaches the unitarity limit $G_0 = 2e^2/h$ at $T = 0$ for symmetric tunneling to the two leads. We have normalized the data to allow for lower values of the zero-temperature conductance caused by asymmetry of the tunnel barriers.

Figs. 5 and 6 are remarkable. The theory was developed to describe the behavior of magnetic impurities in metals, but one could never test the predicted dependence on the parameters $\varepsilon_0$ and $\Gamma$ because for magnetic impurities these parameters are fixed by the composition of the material. On the other hand, in the SET these parameters are controlled by electrode voltages and can be varied continuously. Thus, with the SET we have verified predictions of the renormalization group and scaling theories that have never been tested before.

The data in Figs. 2–6 are for experiments that are very close to equilibrium: the source-drain voltage $V_{ds}$ is kept less than $k_BT/e$. For a magnetic impurity in a metal, it is impossible to disequilibrate the system, but for the SET it is straightforward. Wingreen and Meir [8] have shown that this makes it possible to explore a new kind of Kondo physics. When the Fermi levels of the two leads are substantially different, there are two peaks in the density of states, resulting from the coupling of the spin of the artificial atom to the two leads. As the Fermi levels are separated the excess conductance resulting from the Kondo coupling decreases. This is revealed by measuring the derivative of the current with respect to $V_{ds}$ as a function of $V_{ds}$. Fig. 7 shows the results of such an experiment for the same type of device as studied under equilibrium conditions. The Kondo effect gives rise to a sharp peak at zero bias.

Thus, an electrochemical potential difference between the leads destroys the Kondo coupling. Surprisingly, a magnetic field restores it. In a magnetic field, the peak in the density of states splits into two, one above and one symmetrically below the Fermi energy, one for spin up and one for spin down. When one of these peaks for the left lead lines up in energy with the Fermi energy in the right lead, or vice versa, enhanced conductance is observed. Fig. 7 shows the resulting splitting of the differential conductance peak with magnetic field. Goldhaber-Gordon et al. [9] and Cronenwett et al. [10] find that the magnitude of the splitting in magnetic field is in approximate agreement with theory.

The differential conductance can be displayed in a different way. This is shown in Fig. 8 where $dI/dV$ is plotted on a gray scale as a function of both $V_{ds}$ and $V_e$. The broad diagonal bands correspond to Coulomb charging steps in the conductance. Whenever the combination of gate and drain-source voltages are sufficient to increase the number of electrons on the droplet of the SET such a step occurs and gives rise to a peak in $dI/dV$. However, the sharp feature near
$V_{ds} = 0$ for odd electron number results from Kondo physics. Unlike the Coulomb charging features, the Kondo peak remains at $V_{ds} = 0$ when the gate voltage is changed because the peak in the density of states is tied to the Fermi energy in the leads. The peak splits with magnetic field as expected. We note that the Kondo peak in differential conductance has been observed in other structures, in which the conductance is limited by tunneling through localized states [20–23]. However, the SET is unique in allowing us to vary the critical parameters in a single structure.

4. Outstanding problems

The experimental work has stimulated many theoretical papers [24–27] containing new predictions that should be tested. In particular, Schoeller and Konig [28] have recently reported a new renormalization-group method, which they claim can describe the non-equilibrium properties of a quantum dot strongly coupled to the leads in a SET. Applying this technique they predict the linear (equilibrium) and non-linear conductance. When they fit the data of Goldhaber-Gordon et al. [16] they get very good agreement. But they must use different values of $\epsilon/\Gamma$ than did the experimenters. There is some ambiguity in the original experiments about the choice of the zero of $\epsilon_0$, and this may explain the discrepancy. However, this should be resolved by additional experiments. Most exciting is that these theorists predict the linear and non-linear conductance in the entire range of $\epsilon_0$, including the empty-orbital regime and the mixed-valence regime. Significantly, they predict a peak in the linear conductance as a function of $T$ in the empty orbital regime, for which there is some indication in the experiments of Goldhaber-Gordon et al. However, the conductance at high temperature, especially for large $\epsilon_0$, in the empty orbital regime, is considerably larger than predicted by theory. Schoeller and Konig suggest that this results from low-lying excited states. Schoeller and Konig also predict that in the empty orbital regime there is still a zero-bias peak but that it is not the result of a Kondo resonance. Instead, they say it results from renormalization of the energy level of the quantum dot. They predict that the width of this resonance and its temperature dependence is related to $\Gamma$, a prediction that can be tested.

Glazman et al. [29] have predicted that the Kondo effect persists even when the dot is quite open to the leads. They point out that an important role is played by spin-charge separation in the Kondo effect, and that the spin may be quantized even if the charge is not. They predict a non-monotonic temperature dependence of the conductance in this regime.

The evolution of the differential conductance with magnetic field has been predicted by Costi [31] and by Moore and Wen [30]. Costi, using Wilson’s numerical renormalization group method, predicts that there is a critical magnetic field, below which there is no splitting in the nonlinear conductance. This critical field increases with temperature and results in a peak in the conductance as a function of $T$ for high field but not for low field. Moore and Wen predict more complex behavior. They expect that the
splitting of the nonlinear conductance peak in a magnetic field is not exactly linear in magnetic field. As a result, the overall width of the double peak, they say, should increase more rapidly than linear with field. They also predict unusual asymmetric peaks once $g_\mu H \gg k_BT_K$.

More generally, the nonlinear conductance peak shape should be measured precisely at various temperatures. The variety of theoretical papers on this subject shows the widespread interest in this issue. This has not been done before, in part because the base temperature for most measurements is not much lower than $T_K$ when $\epsilon_0$ is well into the Kondo regime. In particular, the measurements of Goldhaber-Gordon et al. had a lowest electron temperature of about 100 mK, although the base temperature of the refrigerator was about 25 mK.

Another topic that has attracted great theoretical effort is the effects of AC modulation on the Kondo effect [32–39]. Measurements have been made at Delft with relatively large SETs [40]. These show the suppression of the Kondo effect but not the photon sidebands predicted by theory. Clearly this is a fruitful area for study with small SETs.

In summary, we now have solid evidence that the Anderson Hamiltonian provides a quantitative description of single electron transistors, including the subtle Kondo effect. Indeed, it is remarkable that the SET provides a way of tuning the parameters in this many-body Hamiltonian that is not available for natural systems. We expect that studies of non-equilibrium and high frequency Kondo phenomena will provide new and deep insights in the next few years.

Acknowledgements

We acknowledge fruitful discussions with T. Costi, L.I. Glazman, Y. Meir, J. Moore, G. Schöng, H. Schöll and N. Wingreen. This work was supported by the US Army Research Office under contract DAAG 55-98-1-0138 and by the National Science Foundation under grant number DMR-9732579.

References