Playing with virtual photons: Measurement and design of long-range QED forces and torques between microdevices

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Outline

- Zero-point energy
- Casimir effect
- MEMS: Method for Casimir force measurements
- Can we tune/tailor the Casimir force?
  - Thin metallic films
- Torque due to quantum fluctuations
Zero-point energy

First introduced by Planck:

\[ E_n = n\hbar\omega + \frac{\hbar\omega}{2}, n = 0, 1, 2\ldots \]

Zero point energy was needed to reconcile thermodynamics with back body radiation spectrum, when considering a harmonic oscillator in equilibrium at temperature \( T \).

\[ U = \frac{\hbar\omega}{\exp(\hbar\omega/kT) - 1} + \frac{\hbar\omega}{2} \quad \text{with} \quad kT >> \hbar\omega \]

Initially thought that the *zero-point Energy* was of no physical consequence.

\[ g_0(\omega) = \left( \frac{\hbar\omega^2}{\pi^2 c^3} \right) \left( \frac{\hbar\omega}{2} \right) \cdot d\omega \]

\[ E_0 = \int d\omega \cdot g_0(\omega) \rightarrow \infty \]

The total zero-point energy of all modes is infinite!

e.g., P.W. Milonni, Contemporary Physics 1992, V. 33, No. 5, 313
From Maxwell equations

Every electromagnetic wave is equivalent to a harmonic oscillator

\[ A(\mathbf{r}, t) = \alpha(t)A_0(\mathbf{r}) + \alpha^*(t)A_0^*(\mathbf{r}) \]

\[ \nabla^2 A_0(\mathbf{r}) + k^2 A_0(\mathbf{r}) = 0 \quad (k = \omega / c) \]

\[ \mathbf{\Pi}(t) = -\omega^2 \alpha(t) \]

\[ H = \frac{1}{8\pi} \int d\mathbf{r} (E^2 + B^2) = \frac{1}{2} \left( p^2 + \omega^2 q^2 \right) \]

\[ q(t) = \frac{i}{c\sqrt{4\pi}} \left[ \alpha(t) - \alpha^*(t) \right]; \quad p(t) = \frac{k}{\sqrt{4\pi}} \left[ \alpha(t) + \alpha^*(t) \right] \]
Quantization of the EM Field (2)

Hamiltonian for an electromagnetic wave at frequency $\omega$

$H = \hbar \omega \left( a^+ a + \frac{1}{2} \right)$

Uncertainty Principle

$\alpha(t) a(t) \cdot \alpha^*(t) a^+ (t)$

Number of photons at $\omega$

$E_n = \hbar \omega \left( n + \frac{1}{2} \right)$

Number of photons at $\omega$

$\forall n = 0 \quad E_0 = \frac{1}{2} \hbar \omega$

Vacuum is not empty! It is filled with energy (ZERO-POINT ENERGY)

We are thus led to the following conclusions. There exists an attractive force between two metal plates which is independent of the material of the plates as long as the distance is so large that for wave lengths comparable with that distance the penetration depth is small compared with the distance. This force may be interpreted as a zero point pressure of electromagnetic waves.

Although the effect is small, an experimental confirmation seems not unfeasable and might be of a certain interest.

Natuurkundig Laboratorium der N.V. Philips’ Gloeilampenfabrieken, Eindhoven.)
The Casimir Effect: A pioneering paper

Two metallic parallel plates in vacuum: only some modes can exist

\[ E = \sum_{\omega_l} \frac{1}{2} \hbar \omega; \quad \omega_l = c \left( k_x^2 + k_y^2 + \frac{\pi^2}{d^2} l^2 \right)^{1/2} \]

This zero-point energy gives rise to an attractive force:

\[ F = - \frac{\partial E}{\partial d} = - \frac{\pi^2 \hbar c}{240 d^4} L^2 \]
Casimir Force: Intuitive picture

Fluctuating electromagnetic fields associated with virtual photons cause an attraction between metallic plates: Casimir effect (1948)

Only short wavelengths can fit inside the metal plates

Pressure of virtual photons in free space is higher than between plates ⇒ ATTRACTION

1 atmosphere at 100 nanometers separation
Derivation of the Casimir force from zero-point energy consideration

Casimir’s idea was to look at situations where the zero-point Energy might change. He considered two parallel conducting plates.

The allowed frequencies obey:

\[ \omega_{lmn} = \pi c \left( \frac{l^2}{L^2} + \frac{m^2}{L^2} + \frac{n^2}{d^2} \right)^{1/2} \]

The vacuum energy is the sum of the now \textbf{discrete set of modes}

\[ E_0(d) = \sum_{l,m,n} 2 \left( \frac{\hbar \omega_{lmn}}{2} \right) = \sum_{l,m,n} \pi \hbar c \left( \frac{l^2}{L^2} + \frac{m^2}{L^2} + \frac{n^2}{d^2} \right)^{1/2} \]

He considered the \textbf{differences of the vacuum energy}

\[ E_0(d) \propto \sum_{n} \int_{0}^{\infty} \int_{0}^{\infty} dk_x \int_{0}^{\infty} dk_y \left( k_x^2 + k_y^2 + \pi^2 \frac{n^2}{d^2} \right)^{1/2} \]

\[ E_0(\infty) \propto \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} dk_x \int_{0}^{\infty} dk_y \int_{0}^{\infty} dk_z \left( k_x^2 + k_y^2 + k_z^2 \right)^{1/2} \]

\[ U(d) = E_0(d) - E_0(\infty) \]

e.g., P.W. Milonni, Contemporary Physics 1992, V. 33, No. 5, 313
Derivation of the Casimir force by zero-point energy consideration

• This difference diverges due high frequency photons. Physically however we can introduce a cut off since photons of wavelength \( \leq \) than the atomic separation cannot satisfy the boundary conditions. Thus difference does not diverge and we can use the Euler-Maclaurin identity to evaluate it. One finds that:

\[
U(d) = E(d) - E(\infty) = -\frac{\pi^2 \hbar c}{720d^3} L^2
\]

• This gives rise to an attractive force,

\[
F = -\frac{d}{dz} U
\]

varying as the **fourth power** between two metallic plates separated a distance \( z \)

\[
F_c = -\frac{\pi^2 \hbar c}{240} \frac{1}{z^4}
\]

e.g., P.W. Milonni, *Contemporary Physics* 1992, Vol. 33, No. 5, 313
A classical analog of the Casimir effect

Measurements of the Casimir force

- Sparnaay’ 1958
  parallel plates; large uncertainties
- Van Blockland & Overbeek’ 1978
  first clear observation
- Lamoreau’ 1997
  Torsional Pendulum
  first high precision experiment
- Mohideen & Roy’ 1998
  AFM
- Chan, Capasso, et al.’ 2001
  MEMS
- Bressi et al.’ 2003
  parallel plates; high precision

\[ F_{\text{Casimir}} = -\frac{\pi^3 R \hbar c}{360 d^3} \]
Research on Quantum Electrodynamical Forces

Davide Iannuzzi  Mariangela Lisanti  Jeremy Munday

Collaborators: Yuri Barash
Russian Acad. Sci.; Chenogolovka

New student: Marc Romanowski
Previous collaborator: Hobun Chan, Bell Labs
MEMS: MicroElectroMechanical Systems

small, movable structures fabricated with IC technology

Unreleased

Released (HF etch)

Poly Si

Sacrificial Oxide
The Casimir Effect and Moore’s law for MEMS: “nano-consequences” of scaling

\[ P = \frac{F}{L^2} = -\frac{\pi^2 \hbar c}{240d^4} \]

**MEMS (1980-today):**
Moore’s law for MEMS? NEMS might have to contend with the Casimir effect

At 1 \( \mu \text{m} \):

\[ P = 1.3 \text{ mN/m}^2 \]

At 10 nm:

\[ P = 1.3 \text{ atm} \]
Actuation of MEMS by the Casimir force

H. B. Chan, V. A. Aksyuk, R. N. Kleinman, D. J. Bishop, and F. Capasso
Science 291 (2001) 1941
Orders of magnitude

Force sensitivity

\[ \Delta C_{\text{min}} \approx 10^{-6} \text{ pF} \]

\[ \vartheta_{\text{min}} \approx 10^{-7} \text{ rad} \]

Because \( k \approx 10^{-8} \text{ Nm/rad} \)

\[ F_{\text{min}} \approx 10 \text{ pN} \]

Distance sensitivity

\[ \delta_{pz} \text{ Sub-nanometer} \]

We have developed a technique to measure

\[ d_0 \approx 1 \text{ nm} \]
Casimir force measurement with MEMS

Casimir (theoretical curve) (smooth and ideal metals)

Electrostatic (136 mV)
Casimir force: Finite reflectivity and roughness

Casimir (theoretical curve)

— surface roughness (increases the force)

— reflectivity (decreases the force)
Gold surface is an imperfect reflector at UV
Major theoretical advance
Lifshitz Equation, 1956

\[ F = \frac{\hbar}{2\pi c^2} R \int_0^\infty \int_1^\infty p \xi^2 \left\{ \ln \left[ 1 - \frac{(s - p)^2}{(s + p)^2} e^{-2pz\xi / c} \right] + \ln \left[ 1 - \frac{(s - p\varepsilon)^2}{(s + p\varepsilon)^2} e^{-2pz\xi / \omega} \right] \right\} dpd\xi \]

\[ s = \sqrt{\varepsilon - 1 + p^2} \]
\[ \varepsilon = \varepsilon(i\xi) \]

\[ \varepsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \cdot \varepsilon''(\omega)}{\omega^2 + \xi^2} d\omega \]

Contains finite reflectivity corrections
Anharmonic Casimir oscillator

H. B. Chan, V. A. Aksyuk, R. N. Kleinman, D. J. Bishop, and F. Capasso
Anharmonic Casimir oscillator: Frequency shift and hysteresis

d = 112.5 nm

d = 134.3 nm

d = 3.3 μm

A

(µr)
d = 112.5 nm
d = 134.3 nm
d = 3.3 μm

Frequency (Hz)
**Casimir oscillator: A nanometric position sensor?**

Spatial memory: Oscillation amplitude depends on spatial history!
New directions

Can Casimir-Lifshits forces be designed?

Technology applications
New Research Directions

First attempt to switch the Casimir force \textit{in situ}

Measurement of Casimir force using thin metallic films
M. Lisanti, D. Iannuzzi, and F. Capasso, \textit{submitted}

Design of an experiment to detect the torque induced on birefringent plates by quantum fluctuations
Skin depth effect on Casimir force

An “easy example” of tuning to understand what we are up to:
How can we tailor the Casimir force?

Casimir force between “non-ideal” materials
(as opposed to the case of ideally conducting plates)
(plate-sphere configuration)

\[
F = \frac{\hbar}{2\pi c^2} R \int_0^\infty \int_1^\infty \left\{ \ln \left[ 1 - \frac{(s-p)^2}{(s+p)^2} e^{-2p\xi / c} \right] + \ln \left[ 1 - \frac{(s-p\varepsilon)^2}{(s+p\varepsilon)^2} e^{-2p\xi / c} \right] \right\} dp \, d\xi
\]

\[s = \sqrt{\varepsilon - 1 + p^2}\]
\[\varepsilon = \varepsilon(i\xi)\]

\[\varepsilon(i\xi) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \cdot \varepsilon^*(\omega)}{\omega^2 + \xi^2} \, d\omega\]

\[F \leftrightarrow \varepsilon(\omega)\]
Goal: Understanding how the force changes when the metallic layer (Pd) is as thin as the skin-depth

Coated with a thick metallic film

Coated with a metallic film where thickness ~ skin-depth

Quite straightforward… but there are a few technical problems

Mariangela Lisanti
Skin-depth effect on the Casimir force

Main problems:
— surface roughness
— radius of curvature

1\textsuperscript{st} evaporation (thin film) → Meas. of surface roughness → 1\textsuperscript{st} Casimir force meas. (thin film)

2\textsuperscript{nd} evaporation (on top of thin film) → Meas. of surface roughness → 2\textsuperscript{nd} Casimir force meas. (thick film)

— Data with thick film are taken only if the surface roughness is the same

— Same radius of curvature because it is the same sphere (note: Thickness is measured with RBS)
The force decreases due to effect of Skin Depth…
Results: Comparison with theory

...in reasonable agreement with the Lifshitz theory
Birefringent materials: reflection, absorption, transmission depend on orientation

The zero-point energy depends on $\vartheta$, therefore:

$$M = -\frac{\partial E}{\partial \vartheta}$$

V. A. Parsegian and G. H. Weiss, *J. Adhesion* 3 (1972) 259 (non-retarded limit)

Torque induced by quantum fluctuations

In the non-retarded limit and in the limit of slightly birefringent materials:

\[ M = -\frac{\hbar \omega \sin 2\theta}{64\pi^2 d^2} S \]

\[ \bar{\omega} = \int_0^\infty d\omega \frac{-(\varepsilon_1 - \varepsilon_2)^2}{(1 + \varepsilon_2)^2(1 - \varepsilon_2)^2} \ln \left( 1 - \left( \frac{1 - \varepsilon_2}{1 + \varepsilon_2} \right)^2 \right) \]

- Important features:
  - \( M \propto \frac{1}{d^2} \)
  - \( M \propto S \)

- Small separations are necessary
- Area of the surfaces must be relatively large

- Strong signature:
  - \( M \propto \sin 2\theta \)

- Make the experiment difficult (surfaces tend to collapse)
- Can help to distinguish spurious effects

The complete equation?
Torque induced by quantum fluctuations

The complete equation!!!
Quartz or calcite disk (40 µm diameter, 20 µm thick) on top of barium titanate

Torque versus angle for $d = 100$ nm

$M \propto \sin 2\theta$
Quartz or calcite disk (40 µm diameter, 20 µm thick) on top of barium titanate

Torque versus distance at $\theta = 45^\circ$.

It is necessary to keep the two surfaces parallel at distances smaller than a few hundreds of nm.
How can we keep two flat surfaces, free to rotate, parallel at 100 nm?

\[ \varepsilon_1(i\omega) > \varepsilon_3(i\omega) > \varepsilon_2(i\omega) \]

REPULSIVE CASIMIR FORCE!
Mechanical torque on birefringent plates induced by quantum fluctuations: Proposed experimental set-up

The disk floats 100 nm above the plate, free to rotate in response of even very small driving torques

For calcite:
7° in 1 minute
5 minutes to equilibrium (from 45° to 0°)
The experimental technique

\[ d_0 - d_{pz} \ll R \]

\[ \vartheta = k \cdot \left\{ \frac{\varepsilon_0 \pi R}{(d_0 - d_{pz})} \left(V + V_0\right)^2 + F_C \right\} \]

**A priori unknown parameters**

\[ d_0, k \]

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Diagram showing the experimental setup and the parameter relationships.
The experimental technique

\[ \theta = k \frac{\varepsilon_0 \pi R}{(d_0 - d_{pz})} V^2 - 2k \frac{\varepsilon_0 \pi R}{(d_0 - d_{pz})} V_0 V + k \frac{\varepsilon_0 \pi R}{(d_0 - d_{pz})} V_0^2 + kF_C \]

\( d_0, k \)