$Q_0 = -(N + 1)e$ and then begin to increase $V_{ds}$. The Fermi level in the source rises in proportion to $V_{ds}$ relative to the drain, so it also rises relative to the energy levels of the artificial atom. (See the inset to figure 4a.) Current begins to flow when the Fermi energy of the source is raised just above the first quantized energy level of the atom. As the Fermi energy is raised further, higher energy levels in the atom fall below it, and more current flows because there are additional channels for electrons to use for tunneling onto the artificial atom. We measure an energy level by measuring the voltage at which the current increases or, equivalently, the voltage at which there is a peak in the derivative of the current, $dI/dV_{ds}$. (We need to correct for the increase in the energy of the atom with $V_{ds}$, but this is a small effect.) Many beautiful tunneling spectra of this kind have been measured for two-terminal atoms. Figure 4a shows one for a controlled-barrier atom.\textsuperscript{7}

Increasing the gate voltage lowers all the energy levels in the atom by $eV_g$, so that the entire tunneling spectrum shifts with $V_g$, as sketched in figure 3. One can observe this effect by plotting the values of $V_{ds}$ at which peaks appear in $dI/dV_{ds}$. (See figure 4b.) As $V_g$ increases you can see the gap in the tunneling spectrum shift lower and then disappear at the charge-degeneracy point, just as the Coulomb blockade model predicts. You can also see the discrete energy levels of the artificial atom. For the range of $V_{ds}$ shown in figure 4 the voltage is only large enough to add or remove one electron from the atom; the discrete levels above the gap are the excited states of the atom with one extra electron, and those below the gap are the excited states of the atom with one electron missing (one hole). At still higher voltages (not shown in figure 4) one observes levels for two extra electrons or holes and so forth. The charge-degeneracy points are the values of $V_g$ for which one of the energy levels of the artificial atom is degenerate with the Fermi energy in the leads when $V_{ds} = 0$, because only then can the charge of the atom fluctuate.

In a natural atom one has little control over the spectrum of energies for adding or removing electrons. There the electrons interact with the fixed potential of the nucleus and with each other, and these two kinds of interaction determine the spectrum. In an artificial atom, however, one can change this spectrum completely by altering the atom's geometry and composition. For the all-metal atom, which has a high density of electrons, the energy spacing between the discrete levels is so small that it can be ignored. The high density of electrons also results in a short screening length for external electric fields, so electrons added to the atom reside on its surface. Because of this, the electron–electron interaction is always $e^2/C$ (where $C$ is the classical geometrical capacitance), independent of the number of electrons added. This is exactly the case for which the Coulomb blockade model was invented, and it works well: The conductance peaks are perfectly periodic in the gate voltage. The difference between the “ionization potential” and the “electron affinity” is $e^2/C$, independent of the number of electrons on the atom.

In the controlled-barrier atom, as you can see from figure 4, the level spacing is one or two tenths of the energy gap. The conductance peaks are not perfectly periodic in gate voltage, and the difference between ionization potential and electron affinity has a quantum mechanical contribution. I will discuss this contribution a little later in more detail.

In the two-probe atom the electron–electron interaction can be made very small, so that one can in principle reach the limit opposite to that of the all-metal atom. One can find the energy levels of a two-probe atom by measuring the capacitance between its two leads as a function of the voltage between them. When no tunneling occurs, this capacitance is the series combination of the source–atom and atom–drain capacitances. For capacitance measurements, two-probe atoms are made with the insulating layer between the drain and atom so thick that current cannot flow under any circumstances. Whenever the Fermi level in the source lines up with one of the energy levels of the atom, however, electrons can tunnel freely back and forth between the atom and the source. This causes the total capacitance to increase, because the source–atom capacitor is effectively shorted by the tunneling current. The amazing thing about this experiment is that a peak occurs in the capacitance every

**Figure 4**

**Discrete energy levels** of an artificial atom can be detected by varying the drain–source voltage. When a large enough $V_{ds}$ is applied, electrons overcome the energy gap and tunnel from the source to the artificial atom. (See inset of a.) a: Every time a new discrete state is accessible the tunneling current increases, giving a peak in $dI/dV_{ds}$. The Coulomb blockade gap is the region between about $-0.5$ mV and $+0.3$ mV where there are no peaks. b: Plotting the positions of these peaks at various gate voltages gives the level spectrum. Note how the levels and the gap move downward as $V_g$ increases, just as sketched in the lower part of figure 3. (Adapted from ref. 7.)
time a single electron is added to the atom. (See figure 5a.) The voltages at which the peaks occur give the energies for adding electrons to the atom, just as the voltages for peaks in $dI/dV_{ds}$ do for the controlled-barrier atom or for a two-probe atom in which both the source–atom barrier and the atom–drain barrier are thin enough for tunneling. The first peak in figure 5a corresponds to the one-electron artificial atom.

Figure 5b shows how the energies for adding electrons to a two-probe atom vary with a magnetic field perpendicular to the GaAs layer. In an all-metal atom the levels would be equally spaced, by $e^2/C$, and would be independent of magnetic field because the electron–electron interaction completely determines the energy. By contrast, the levels of the two-probe atom are irregularly spaced and depend on the magnetic field in a systematic way. For the two-probe atom the fixed potential determines the energies at zero field. The level spacings are irregular because the potential is not highly symmetric and varies at random inside the atom because of charged impurities in the GaAs and AlGaAs. It is clear that the electron–electron interactions that are the source of the Coulomb blockade are not always so important in the two-probe atom as in the all-metal and controlled-barrier atoms. Their relative importance depends in detail on the geometry.\textsuperscript{5}

**Artificial atoms in a magnetic field**

Level spectra for natural atoms can be calculated theoretically with great accuracy, and it would be nice to be able to do the same for artificial atoms. No one has yet calculated an entire spectrum, like that in figure 4a. However, for a simple geometry we can now predict the charge-degeneracy points, the values of $V_e$ corresponding to conductance peaks like those in figure 2. From the earlier discussion it should be clear that in such a calculation one must take into account the electron’s interactions with both the fixed potential and the other electrons.

The simplest way to do this is with an extension of the Coulomb blockade model.\textsuperscript{11–13} It is assumed, as before, that the contribution to the gap in the tunneling spectrum from the Coulomb interaction is $e^2/C$ no matter how many electrons are added to the atom. To account for the discrete levels one pretends that once on the atom, each electron interacts independently with the fixed potential. All one has to do is solve for the energy levels of a single electron in the fixed potential that creates the artificial atom and then fill those levels in accordance with the Pauli exclusion principle. Because the electron–electron interaction is assumed always to be $e^2/C$, this is called the constant-interaction model.

Now think about what happens when one adds electrons to a controlled-barrier atom by increasing the gate voltage while keeping $V_{ds}$ just large enough so one can measure the conductance. When there are $N - 1$ electrons on the atom the $N - 1$ lowest energy levels are filled. The next conductance peak occurs when the gate voltage pulls the energy of the atom down enough that the Fermi level in the source and drain becomes degenerate with the $N$th level. Only when an energy level is degenerate with the Fermi energy can current flow; this is the condition for a conductance peak. When $V_g$ is increased further and the next conductance peak is reached, there are $N$ electrons on the atom, and the Fermi level is degenerate with the $(N + 1)$-th level. Therefore to get from one peak to the next the Fermi energy must be raised by $e^2/C + (E_{N+1} - E_N)$, where $E_N$ is the energy of the $N$th level of the atom. If the energy levels are closely spaced the Coulomb blockade result is recovered, but in general the level spacing contributes to the energy between successive conductance peaks.

It turns out that we can test the results of this kind of calculation best if a magnetic field is applied perpendicular to the GaAs layer. For free electrons in two dimensions, applying the magnetic field results in the spectrum of Landau levels with energies $(n + 1/2)\hbar \omega_c$, where the cyclotron frequency is $\omega_c = eB/m^*c$, and $m^*$ is the effective mass of the electrons. In the controlled-barrier atom and the two-probe atom, we expect levels that behave like Landau levels at high fields, with energies that increase linearly in $B$. This behavior occurs because when the field is large enough the cyclotron
radius is much smaller than the size of the electrostatic potential well that confines the electrons, and the electrons act as if they were free. Levels shifting proportionally to $B$, as expected, are seen experimentally. (See figure 5b.)

To calculate the level spectrum we need to model the fixed potential, the analog of the potential from the nucleus of a natural atom. The simplest choice is a harmonic oscillator potential, and this turns out to be a good approximation for the controlled-barrier atom. Figure 6a shows the calculated level spectrum as a function of magnetic field for noninteracting electrons in a two-dimensional harmonic oscillator potential. At low fields the energy levels dance around wildly with magnetic field. This occurs because some states have large angular momentum and the resulting magnetic moment causes their energies to shift up or down strongly with magnetic field. As the field is increased, however, things settle down. For most of the field range shown there are four families of levels, two moving up, the other two down. At the highest fields there are only two families, corresponding to the two possible spin states of the electron.

Suppose we measure, in an experiment like the one whose results are shown in figure 2, the gate voltage at which a specific peak occurs as a function of magnetic field. This value of $V_g$ is the voltage at which the $N$th energy level is degenerate with the Fermi energy in the source and drain. A shift in the energy of the level will cause a shift in the peak position. The blue line in figure 6a is the calculated energy of the 39th level (chosen fairly arbitrarily for illustration purposes), so it gives the prediction of the constant-interaction model for the position of the 39th conductance peak. As the magnetic field increases, levels moving up in energy cross those moving down, but the number of electrons is fixed, so electrons jump from upward-moving filled levels to downward-moving empty ones. The peak always follows the 39th level, so it moves up and down in gate voltage.

Figure 6b shows a measurement of $V_g$ for one conductance maximum, like one of those in figure 2, as a function of $B$. The behavior is qualitatively similar to that predicted by the constant-interaction model: The peak moves up and down with increasing $B$, and the frequency of level crossings changes at the field where only the last two families of levels remain. However, at high $B$ the frequency is predicted to be much lower than what is observed experimentally. While the constant-interaction model is in qualitative agreement with experiment, it is not quantitatively correct.

To anyone who has studied atomic physics, the constant-interaction model seems quite crude. Even the simplest models used to calculate energies of many-electron atoms determine the charge density and potential self-consistently. One begins by calculating the charge density that would result from noninteracting electrons in the fixed potential, and then one calculates the effective potential an electron sees because of the fixed potential and the potential resulting from this charge density. Then one calculates the charge density again. One does this repeatedly until the charge density and potential are self-consistent. The constant-interaction model fails because it is not self-consistent. Figure 6c shows the results of a self-consistent calculation for the controlled-barrier atom. It is in good agreement with experiment—much better agreement than the constant-interaction model gives.

Conductance line shapes

In atomic physics, the next step after predicting energy levels is to explore how an atom interacts with the electromagnetic field, because the absorption and emission of photons teaches us the most about atoms. For artificial atoms, absorption and emission of electrons plays this role, so we had better understand how this process works. Think about what happens when the gate voltage in the controlled-barrier atom is set at a conductance peak, and an electron is tunneling back and forth between the atom and the leads. Since the electron spends only a finite time $\tau$ on the atom, the uncertainty principle tells us that the energy level of the electron has a width $\hbar/\tau$. Furthermore, since the probability of finding the electron on the atom decays as $e^{-\lambda \tau}$, the level will have a Lorentzian line shape.

This line shape can be measured from the transmission probability spectrum $T(E)$ of electrons with energy $E$ incident on the artificial atom from the source. The spectrum is given by

$$T(E) = \frac{\Gamma^2}{\Gamma^2 + (E - E_N)^2}$$

where $\Gamma$ is approximately $\hbar/\tau$ and $E_N$ is the energy of the $N$th level. The probability that electrons are transmitted from the source to the drain is approximately proportionally to the conductance $G$. In fact, $G = (e^2/h)T$, where $e^2/h$ is the quantum of conductance. It is easy to show that one must have $G < e^2/h$ for each of the barriers separately to observe conductance resonances. (An equivalent argument is used to show that electrons in a disordered conductor are localized for $G < e^2/h$. See, for example, the article by Boris L. Al'tshuler and Patrick A. Lee in PHYSICS TODAY, December 1988, page 36.) This condition is equivalent to requiring that the separation of the levels is greater than their width $\Gamma$.

Like any spectroscopy, our electron spectroscopy of artificial atoms has a finite resolution. The resolution is determined by the energy spread of the electrons in the source, which are trying to tunnel into the artificial atom. These electrons are distributed according to the Fermi–Dirac function,

$$f(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

where $E_F$ is the Fermi energy. The tunneling current is given by

$$I = \int \frac{e}{h} T(E) \left[ f(E) - f(E - eV_{ds}) \right] dE$$

Equation 4 says that the net current is proportional to the probability $f(E)T(E)$ that there is an electron in the source with energy $E$ and that the electron can tunnel between the source and drain minus the equivalent probability for electrons going from drain to source. The best resolution is achieved by making $V_{ds} \ll kT$. Then $[f(E) - f(E - eV_{ds})] \approx eV_{ds} dE/dE$, and $I$ is proportional to $V_{ds}$, so the conductance is $I/V_{ds}$.

Figure 2 shows that equations 2–4 describe the experiments well: At low $V_{gs}$, where $\Gamma$ is much less than $kT$, the shape of the conductance resonance is given by the resolution function $dE/dE$. But at higher $V_{gs}$ one sees the Lorentzian tails of the natural line shape quite clearly. The width $\Gamma$ depends exponentially on the height and width of the potential barrier, as is usual for tunneling. The height of the tunnel barrier decreases with $V_{gs}$, which is why the peaks become broader with increasing $V_{gs}$. Just as we have control over the level spacing in artificial atoms, we also can control the coupling to the leads and therefore the level widths. It is clear why the present generation of artificial atoms show unusual behavior only at low temperatures: When $kT$ becomes comparable to
the energy separation between resonances, the peaks overlap and the features disappear.

Applications

The behavior of artificial atoms is so unusual that it is natural to ask whether they will be useful for applications to electronics. Some clever things can be done. Because of the electron-electron interaction, only one electron at a time can pass through the atom. With devices like the "turnstile" device$^{16,17}$ shown on the cover of this issue the two tunnel barriers can be raised and lowered independently. Suppose the two barriers are raised and lowered sequentially at a radio or microwave frequency $\nu$. Then, with a small source-drain voltage applied, an electron will tunnel onto the atom when the source-atom barrier is low and off it when the atom-drain barrier is low. One electron will pass in each time interval $\nu^{-1}$, producing a current ev. Other applications, such as sensitive electrometers, can be imagined.$^{9,18}$ However, the most interesting applications may involve devices in which several artificial atoms are coupled together to form artificial molecules$^{16,17,19}$ or in which many are coupled to form artificial solids. Because the coupling between the artificial atoms can be controlled, new physics as well as new applications may emerge. The age of artificial atoms has only just begun.

References


